Topic 4 Waves
Oscillation

A regular variation in magnitude or position about a central point
Displacement - $x$

The distance and direction from the equilibrium position.
Amplitude - A

The maximum displacement from the equilibrium position.
Period - $T$

The time taken (in seconds) for one complete oscillation. It is also the time taken for a complete wave to pass a given point.
Frequency - f

The number of oscillations in one second. Measured in Hertz.

50 Hz = 50 vibrations/waves/oscillations in one second.
Period and frequency

Period and frequency are reciprocals of each other

\[ f = \frac{1}{T} \quad T = \frac{1}{f} \]
Phase difference

• is the time difference or phase angle by which one wave/oscillation leads or lags another.

180° or π radians
Phase difference

\[ \varphi = \frac{\text{shift}}{T} \times 360^\circ \]

90° or \(\pi/2\) radians
Let’s watch a video

Simple Harmonic motion
Simple harmonic motion (SHM)

- periodic motion in which the restoring force/acceleration is proportional and in the opposite direction to the displacement
Simple harmonic motion (SHM)

\[ \mathbf{a} - \alpha \mathbf{x} \]
Simple harmonic motion (SHM)

- Hooke’s law leads to SHM

\[ F = -kx \]

(\(a\ \alpha\ -x\))
A graph of the motion will have this form.
A graph of the motion will have this form.
\( y = \text{sun}(x) \)
Where is velocity at a max.?
Maximum velocity?

- When $x = 0$

- At this point the acceleration is zero (no resultant force at the equilibrium position).
Plotting displacement and velocity versus time
Where is acceleration at a max.?
Maximum acceleration?

- When $x = +/\!\!/ - x_0$

- Here the velocity is zero
Acceleration against time

displacement

acceleration
4.1 Period of an oscillating mass

\[ T = 2\pi \sqrt{\frac{m}{k}} \]
Where is the kinetic energy maximum?

Where is the potential energy maximum?
No Air Friction
Kinetic energy is maximum at $x = 0$
Potential energy is maximum at $x = \pm x_o$
Natural frequency

All objects have a natural frequency that they prefer to vibrate at.
Resonance

If the frequency of the external force is equal to the natural frequency we get resonance

YouTube - Ground Resonance - Side View

YouTube - breaking a wine glass using resonance

http://www.youtube.com/watch?v=6ai2QFxStxo&feature=elmfu
The Singing Tree!
SHM Graphs

Derivative Form

Position

\[ r(t) \]

Velocity

\[ v(t) = \frac{dr}{dt} \]

Acceleration

\[ a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2} \]
A graph to show the relationship between displacement, velocity and acceleration of a body displaying simple harmonic motion.

Each of the three Y values would have a different scale and a different unit!
4.2 Travelling Waves

All travelling waves carry energy, but generally the medium through which they travel will not be permanently disturbed.

It took centuries of careful observation to recognize patterns and trends to show that light, sound, disturbances in water, and earthquakes all exhibited wave behavior.
4.2 Travelling waves

We say a wave moving away from its source is *propagating*.

Any substance through which a wave passes is a *medium*.

If a disturbance is not continuous, we say it is a *pulse* rather than a wave.
4.2 Travelling waves

*Mechanical waves* involve oscillating masses and thus need material through which to travel. (ex. sound)

*Electromagnetic waves* are able to travel through space. (ex. Light, radio, x-rays)
The Electromagnetic Spectrum

Electromagnetic waves transport energy through empty space, stored in the propagating electric and magnetic fields.

Magnetic field variation is perpendicular to electric field.

A single-frequency electromagnetic wave exhibits a sinusoidal variation of electric and magnetic fields in space.
The Electromagnetic Spectrum

Penetrates Earth’s Atmosphere?

<table>
<thead>
<tr>
<th>Radiation Type</th>
<th>Radio</th>
<th>Microwave</th>
<th>Infrared</th>
<th>Visible</th>
<th>Ultraviolet</th>
<th>X-ray</th>
<th>Gamma ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (m)</td>
<td>$10^3$</td>
<td>$10^{-2}$</td>
<td>$10^{-5}$</td>
<td>$0.5 \times 10^{-6}$</td>
<td>$10^{-8}$</td>
<td>$10^{-10}$</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

Approximate Scale of Wavelength

- Buildings
- Humans
- Butterflies
- Needle Point
- Protozoans
- Molecules
- Atoms
- Atomic Nuclei

Frequency (Hz)

- $10^4$
- $10^8$
- $10^{12}$
- $10^{15}$
- $10^{16}$
- $10^{18}$
- $10^{20}$

Temperature of objects at which this radiation is the most intense wavelength emitted

- $1 \text{ K}$: $-272 \text{ °C}$
- $100 \text{ K}$: $-173 \text{ °C}$
- $10,000 \text{ K}$: $9,727 \text{ °C}$
- $10,000,000 \text{ K}$: $-10,000,000 \text{ °C}$
Transverse waves

In a transverse wave, each part of the medium oscillates \textit{perpendicularly} to the direction in which the wave is transferring energy.

Examples?

Light (all EM waves), water waves, waves on a string.
Longitudinal Waves

In a *longitudinal wave* (compression waves), each part of the medium oscillates parallel to the direction in which the wave is transferring energy.

Examples:
Sound waves, part of earthquake waves.
Longitudinal Waves

Compressions are where the spring compresses.

Rarefactions are where the spring stretches.
Representing waves graphically

Wavelength
Period
Frequency
Wave speed $v = f \lambda$

For light: $c = f \lambda$
Wavelength, frequency, period and wave speed

Wavelength: the shortest distance between 2 points
Period (T): the time
Frequency
Wave speed \( v = f\lambda \)
The electromagnetic spectrum

‘Light’ is just the name we give to EM waves that human beings can detect with our eyes.
The nature of the EM spectrum

EM waves puzzled science for a long time because all other waves need a medium in which to travel.

We now understand that EM waves are propagated by linked oscillating magnetic and electric fields.

However, EM waves also have some properties that can only be explained by thinking of them as ‘particles’ (not waves).

These particles are photons! Each photon can transfer a specific amount of energy dependent on its frequency.
Heinrich Hertz

German physicist known for proving experimentally, in 1886, that EM waves (radio) could be produced, transmitted, and detected elsewhere.

“Broke your hand waving?”

I bet that Hertz.
The nature of sound waves

When the surface of a solid vibrates, it will disturb the air (or another medium) that surrounds it and produce a series of compressions and rarefactions that travel away from the surface as a longitudinal wave.

If we can hear the waves - we call them sound!

- The audible range is ~ 20 Hz - 20 kHz
The speed of sound

\[ v = 343\text{ms}^{-1} \text{ at } 20^\circ\text{C} \]

Why does the temperature matter?
4.3 Wave Characteristics

A **wavefront** is a line joining adjacent points moving in phase.

A **ray** is a line pointing in the direction in which the wave energy is being transferred.

- A ray is always normal to a wavefront.
4.3 Wave Characteristics

Planar wavefronts are parallel to one another, movement is represented by parallel rays.

Radial wavefronts that travel long distance from their source, become very nearly parallel and will approximate planar wavefronts.

Ex. Light from our sun
Amplitude and Intensity

**Intensity** is the wave power passing perpendicularly through a unit area.

Intensity \((\text{Wm}^{-2}) = \frac{\text{Power}}{\text{Area}}\)
Intensity and amplitude

This means if you double the amplitude of a wave, its intensity quadruples!

\[ I = kA^2 \]

If amplitude = 2A,
new intensity = \( k(2A)^2 \)
new intensity = \( k4A^2 \)
Surfers know this!
Intensity

As circular wavefronts travel away from a point source, their circumferences increase, so that the same amount of energy becomes spread out over a longer length. This leads to a decrease in amplitude

\[ I \propto A^2 \]

A represents amplitude, not area.
Inverse Square Law

For waves that spread evenly in three dimensions from a point source without any loss in energy, their intensity, $I$, is inversely proportional to the distance from the source, $x$, squared.
Inverse Square Law

\[ I \propto x^{-2} \]
The principle of superposition states that at any moment the overall displacement at any point will be the vector sum of all the individual wave displacements.
Constructive and destructive interference

When two waves of the same frequency superimpose, we can get constructive interference or destructive interference.
Superposition

In general, the displacements of two (or more) waves can be added to produce a resultant wave. (Note, displacements can be negative)

The principle of superposition will help us to explain interference patterns later in this chapter.
Reflection of pulses

A rope with a fixed end will be reflected by a wall, creating a phase change of 180°.
- This is due to Newton’s 3rd Law.

The rope exerts an upward force on the wall. The wall exerts an equal but opposite force on the rope. This results in a negative displacement with the rope moving to the left.
Polarization

A wave may be described as **plane polarized** when it is only oscillating in one plane.

A transverse wave is polarized if all the oscillations transferring the wave’s energy are in the same plane - the **plane of polarization**.
A polarized wave has oscillations that are perpendicular to the direction of travel. This means it is **impossible** for longitudinal waves to be polarized.
Polarization

EM waves are mostly emitted during random, unpredictable processes, so we expect them to oscillate in random directions (or be unpolarized).
Polarization by absorption

Unpolarized light may become polarized by passing it through a *polarizer* - a sheet of material in which long molecular chains are aligned in one direction.

Components of e-fields which are perpendicular to the molecules are transmitted; while parallel are absorbed.
Edwin Land, a 19 year old Harvard undergraduate, invented this material - it was further refined and given the name Polaroid.
Polarization by absorption

The second filter is called an **analyzer** because it can be rotated to analyse the light and determine if it is polarized and, if so, in what direction.
Malus’s Law

The transmitted intensity is proportional to the square of the amplitude of the e-field.

\[ I = I_0 \cos^2 \theta \]
Polarization by reflection

Unpolarized light may reflect off a surface and become polarized:

- The *plane of polarization* will be parallel to the reflecting surface.
- We commonly refer to this as glare!

This ‘glare’ is reduced through the use of polarized lenses.
Iceland Spar (sunstone)

It is thought that as much as 1000 years ago Vikings use Iceland spar as a navigation tool because it’s effect on rays of light allow one to determine the position of the sun, even behind clouds.

Unpolarized light passing through Iceland spar will result in a double refraction as seen above.
Particles or a wave?

Thomas Young and Augustin-Jean Fresnel did much to advance the wave theory of light. This required a drastic shift from the Newtonian view - which required much bravery.

The phenomenon of polarization which helped Young to describe light as a transverse wave. Later Maxwell would propose the EM nature of light.
4.4 Wave Behaviour

Waves interact with media and each other in a number of ways that can be unexpected and useful.
Reflection

When a wave meets a boundary between two different media, usually some or all of the waves will be reflected back.

Under certain circumstances some waves may pass into or through the second medium - we say there was some transmission of the waves.

- Think of transparent materials such as glass.
Reflection

A mirror
Reflection

A line is **normal** when it is perpendicular to a surface.
We say a wave is **incident** (falling on) upon the surface of the mirror.
Reflection

Angle of incidence
Reflection

Angle of incidence

normal
Angle of incidence = Angle of reflection
What color is a mirror?
Mirrors on the moon!
4.4 Mirror Island
The mug trick!
The mug trick!
Why does this happen?
Marching soldiers!
Light waves

Light slows down as it goes from air to glass/water.
Ripple tank

- Ripple Tank Simulation
Snell’s law

There is a relationship between the speed of the wave in the two media and the angles of incidence and refraction.
Snell’s law

\[
\frac{\text{speed in substance 1}}{\text{speed in substance 2}} = \frac{\sin \theta_1}{\sin \theta_2}
\]
Snell’s law

In the case of light only, we usually define a quantity called the index of refraction for a given medium as

\[ n = \frac{c}{c_m} = \frac{\sin \theta_1}{\sin \theta_2}, \]

where \( c \) is the speed of light in a vacuum and \( c_m \) is the speed of light in the medium.
Snell’s law

Thus for two different media

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}
\]
Thus for two different media

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = \frac{1}{n_2}
\]